450

VOLUME 80 SEPARATE No. 450

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

JUNE, 1954

CONTINUOUS DEEP BEAMS

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Presented at
Atlantic City Convention
June 14-19, 1954

STRUCTURAL DIVISION

{Discussion open until October 1, 1954}

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Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

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This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

CONTINUOUS DEEP BEAMS

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SYNOPSIS

A rigorous solution for a continuous deep girder of equal spans subjected to all types of symmetrical loads at the top, bottom, or its own weight, is presented. The condition of no displacement at supports is specified. The distribution of bending stresses is illustrated by graphs. The difference between this solution and the earlier solution by Dischinger is discussed.

INTRODUCTION

A deep girder may be defined as a beam whose bending stresses deviate appreciably from the straight line distribution assumed in the elementary beam theory. Girders with a height-to-span ratio of one-half or more fall in this category. A deep girder supported on an infinite number of equally spaced supports was first investigated by Franz Dischinger[3]. The results of his analysis have been made available in a practical and concise form[4] for structural engineers. In the same paper, Dischinger also proposed an approximate solution for simply supported girders. It is believed that more precise and complete solutions for simple girders are available now[5, 6].

The Airy's stress function used by Dischinger was expressed in terms of Fourier Series. The boundary conditions at the top and bottom edges of the girder were satisfied. It can be shown that this solution implies a longitudinal displacement of supports, depending on loading conditions, height-to-span ratio, and elastic properties of the girder, and proportional to its distance from the vertical line of symmetry of the girder except when the girder is

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3. "Beitrag zur Theorie der Halbscheibe und des wandartigen Balkens" by Franz Dischinger, International Association for Bridge and Structural Engineering Publications, Vol. 1, 1932
4. "Design of Deep Girders", Concrete Information No. ST-66, Portland Cement Association
5. "Analysis of Deep Beams", by H. D. Conway, L. Chow, and G. W. Morgen, Journal of Applied Mechanics, ASME Vol. 18, No. 2, June, 1951. See also "Stresses in Deep Beams", by L. Chow, H. D. Conway, and G. Winter, ASCE Proceeding Separate No. 127, May, 1952
6. "The Theory of Girder Walls with Special Reference to Reinforced Concrete Design", by H. L. Uhlmann, The Structural Engineer, Vol. XXX, No. 8, August, 1952, P. 172

loaded at its lower edge. In other words, the solution is exact if the girder is mounted on roller supports with particular loading.

This paper offers a similar solution of a deep girder continuous over an infinite number of equally spaced, but fixed supports, subjected to either concentrated, partial or full uniform loads at the top or bottom edge of the girder. A girder subjected to its own weight is also investigated. The bending stresses at mid-span and at supports, and the shear stresses at interesting points are computed, illustrated and compared.

The Basic Theory

It is assumed that the girder is made of homogeneous, isotropic and perfectly elastic material. The load across the width of the girder is assumed to be uniform so that the girder may be treated as a problem of plane stress. For simplicity, the width of the girder is taken as unity in the following analysis.

The stress distribution in the girder may be derived from a single stress function F(x, y) which satisfies the bi-harmonic equation:

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 . \tag{1}$$

The effect of body weight is excluded here. It will be treated later as a separate case. The stress function for a continuous girder of equal spans 2L, subjected to a periodic loading of period 2L may be written as follows:

$$\begin{aligned} F(x,y) = {} & Ax^2 + By^2 \\ & + \sum_{n=1}^{\infty} \cos\frac{n\pi}{L}x \left(C_1 \cosh\frac{n\pi}{L}y + C_2 \sinh\frac{n\pi}{L}y \right. \\ & \left. + C_3 y \cosh\frac{n\pi}{L}y + C_4 y \sinh\frac{n\pi}{L}y \right). \end{aligned} \tag{2}$$

Letting α denote $\frac{n\pi}{L}$, the stresses will be

$$\begin{aligned} \sigma_x = \frac{\partial^2 F}{\partial y^2} = {} & 2B + \sum_{n=1}^{\infty} \cos\alpha x \left[C_1 \alpha^2 \cosh\alpha y + C_2 \alpha^2 \sinh\alpha y \right. \\ & + C_3 \alpha (2\sinh\alpha y + \alpha y \cosh\alpha y) \\ & \left. + C_4 \alpha (2\cosh\alpha y + \alpha y \sinh\alpha y) \right] , \end{aligned} \tag{3}$$

$$\begin{aligned} \sigma_y = \frac{\partial^2 F}{\partial x^2} = {} & 2A - \sum_{n=1}^{\infty} \alpha^2 \cos\alpha x \left[C_1 \cosh\alpha y + C_2 \sinh\alpha y \right. \\ & \left. + C_3 y \cosh\alpha y + C_4 y \sinh\alpha y \right] , \end{aligned} \tag{4}$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} = \sum_{n=1}^{\infty} \alpha \sin\alpha x \left[C_1 \alpha \sinh\alpha y + C_2 \alpha \cosh\alpha y + C_3 (\cosh\alpha y + \alpha y \sinh\alpha y) + C_4 (\sinh\alpha y + \alpha y \cosh\alpha y) \right] , \tag{5}$$

where A, B, C_1, C_2, C_3, C_4 are constants to be determined by proper boundary conditions.

It is easy to verify that each term in eqs. (2) satisfies the bi-harmonic equation (1). If the constants are determined by satisfying the stresses at the boundary and zero displacement condition at the supports, the pattern of stress distribution is determined.

CASE(I) Partial Uniform Loads Applied At Top Edge Of The Girder (Fig. 1)

Uniformly distributed load q is applied over a length of "2d" at the center of each span. The following boundary conditions must be satisfied:

$$\left.\begin{aligned} y = +H, \quad \tau_{xy} = 0, \quad \sigma_y &= -\frac{qd}{L} - \frac{2q}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\alpha d \cos\alpha x ; \\ y = -H, \quad \tau_{xy} = 0, \quad \sigma_y &= -\frac{qd}{L} - \frac{2qd}{\pi c} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\alpha c \cos\alpha x . \end{aligned}\right\} \tag{6}$$

In addition,

$$u = \frac{1}{E} \int_{-L}^{+L} (\sigma_x - \nu \sigma_y)\, dx = 0 .$$

where ν is Poisson's ratio and u is the longitudinal displacement. The last boundary condition specifies that the distance between centerline of supports must remain unchanged. The constants are obtained by substituting eqs. (3), (4), and (5) into eq. (6). They are:

$$\left.\begin{aligned} C_1 &= \frac{q}{n\pi\alpha^2} \left[\sin\alpha d + (-1)^n \frac{d}{c} \sin\alpha c \right] K_1 , \\ C_2 &= \frac{q}{n\pi\alpha^2} \left[\sin\alpha d - (-1)^n \frac{d}{c} \sin\alpha c \right] K_2 , \\ C_3 &= \frac{q}{n\pi\alpha} \left[-\sin\alpha d + (-1)^n \frac{d}{c} \sin\alpha c \right] K_3 , \\ C_4 &= \frac{q}{n\pi\alpha} \left[-\sin\alpha d - (-1)^n \frac{d}{c} \sin\alpha c \right] K_4 , \\ A &= -\frac{qd}{2L} \quad \text{and} \quad B = -\frac{\nu q d}{2L} . \end{aligned}\right\} \tag{7}$$

where,

$$K_1 = \frac{\sinh\alpha H + \alpha H \cosh\alpha H}{\sinh\alpha H \cosh\alpha H + \alpha H} ,$$

$$K_2 = \frac{\cosh\alpha H + \alpha H \sinh\alpha H}{\sinh\alpha H \cosh\alpha H - \alpha H} ,$$

$$K_3 = \frac{\cosh\alpha H}{\sinh\alpha H \cosh\alpha H - \alpha H} ,$$

$$K_4 = \frac{\sinh\alpha H}{\sinh\alpha H \cosh\alpha H + \alpha H} .$$

(8)

The terms K_1, K_2, K_3, and K_4 will appear repeatedly in later equations.

It is of interest to note that the constant B appearing in the bending stresses is due to the fixed support condition. If the fixity is removed, B will be zero.

Uniform Load (Fig. 2)

By setting d equal to L in eqs. (7), the solution for uniform load over the entire girder is obtained. The constants in eqs. (3), (4) and (5) become:

$$C_1 = (-1)^n \frac{q \sin\alpha c}{\alpha^3 c} K_1 ,$$

$$C_2 = (-1)^{n+1} \frac{q \sin\alpha c}{\alpha^3 c} K_2 ,$$

$$C_3 = (-1)^n \frac{q \sin\alpha c}{\alpha^2 c} K_3 ,$$

$$C_4 = (-1)^{n+1} \frac{q \sin\alpha c}{\alpha^2 c} K_4 ,$$

$$A = -\frac{q}{2} \quad \text{and} \quad B = -\frac{\nu q}{2} .$$

(9)

Concentrated Load (Fig. 3)

If d is set equal to c in eqs. (7), the constants for eqs. (3), (4) and (5) are:

$$C_1 = \left[1 + (-1)^n\right] \frac{q \sin\alpha c}{n\pi\alpha^2} K_1 ,$$

$$C_2 = \left[1 - (-1)^n\right] \frac{q \sin\alpha c}{n\pi\alpha^2} K_2 ,$$

$$C_3 = \left[-1 + (-1)^n\right] \frac{q \sin\alpha c}{n\pi\alpha} K_3 ,$$

$$C_4 = \left[-1 - (-1)^n\right] \frac{q \sin\alpha c}{n\pi\alpha} K_4 ,$$

$$A = -\frac{qc}{2L} \quad \text{and} \quad B = -\frac{\nu q c}{2L} .$$

(10)

When both d and c are small in comparison with the span length (say, one-tenth of the span), then for all practical purposes, we may call this as a case of concentrated load.

Point Loads with Point Supports (Fig. 4)

If both d and c in eqs. (7) approach zero, but the products 2qc and 2qd remain a constant P, it becomes the case of point loads at center of spans. It may be verified that the constants in eqs. (3), (4) and (5) are:

$$\left.\begin{aligned} C_1 &= \left[1+(-1)^n\right]\frac{P}{2n\pi\alpha}K_1, \\ C_2 &= \left[1-(-1)^n\right]\frac{P}{2n\pi\alpha}K_2, \\ C_3 &= \left[-1+(-1)^n\right]\frac{P}{2n\pi}K_3, \\ C_4 &= \left[-1-(-1)^n\right]\frac{P}{2n\pi}K_4, \\ A &= -\frac{P}{4L} \quad \text{and} \quad B = -\frac{\nu P}{4L}. \end{aligned}\right\} \tag{11}$$

Other loading variation can be obtained by superposition. Fig. 11, for instance, shows two cases of partial uniform load applied over the supports.

CASE(II) Partial Uniform Loads Applied At Bottom Edge Of The Girder (Fig. 5)

The stresses are still given by eqs. (3), (4) and (5), but the constants A, B, C_1, C_2, C_3 and C_4 in these equations are evaluated from the following boundary conditions:

$$\left.\begin{aligned} y=+H \qquad & \tau_{xy}=0, \qquad \sigma_y=0; \\ y=-H \qquad & \tau_{xy}=0, \\ & \sigma_y=-\frac{2q}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[(-1)^n\frac{d}{c}\sin\alpha c-\sin\alpha d\right]\cos\alpha x; \end{aligned}\right\} \tag{6a}$$

and,

$$\int_{-L}^{L}(\sigma_x-\nu\sigma_y)\,dx=0.$$

These constants are found to be:

$$\left.\begin{aligned} C_1 &= \frac{q}{n\pi\alpha^2}\left[-\sin\alpha d + (-1)^n \frac{d}{c}\sin\alpha c\right] K_1, \\ C_2 &= \frac{q}{n\pi\alpha^2}\left[\sin\alpha d - (-1)^n \frac{d}{c}\sin\alpha c\right] K_2, \\ C_3 &= \frac{q}{n\pi\alpha}\left[-\sin\alpha d + (-1)^n \frac{d}{c}\sin\alpha c\right] K_3, \\ C_4 &= \frac{q}{n\pi\alpha}\left[\sin\alpha d - (-1)^n \frac{d}{c}\sin\alpha c\right] K_4, \\ A &= B = 0. \end{aligned}\right\} \qquad (12)$$

By varying the dimension d, various solutions similar to those discussed in Case (I) may be obtained.

It is interesting to note that, if the last condition--condition of fixed supports--in eqs (6a) was omitted, the results in eqs. (12) will not be altered. In other words, when the load is applied at the bottom edge of the girder, no longitudinal displacement will be experienced by the supports. Consequently the same stress pattern will result in this case, whether the girder is supported by roller supports, or fixed supports. Without specifying the condition at supports, the identical results were first obtained by Dischinger (3) for concentrated and fully uniform loads. However, it is worthy to point out that, if the load is applied at the elevation other than the lower edge, different stress patterns will result for the cases of roller and fixed supports.

Comparing the constants C's in eqs. (12) with those in eqs. (7), it is noted that they are not identical. However, if d is set equal to L, the first term in those constants vanishes, and the corresponding C's in eqs. (12) and eqs. (7) become identical and have values as shown in eqs. (9). In other words, when the load is uniformly distributed either at the top or bottom edge of the entire span, both bending and shear stresses will remain the same for the girder on rollers; and shear stresses will remain the same for the girder on fixed supports. Further investigation shows that the shear stresses in this case is independent of the load elevations.

CASE (III) Continuous Girder Subjected To Its Own Weight (Fig. 6)

The stress function in this case may be chosen as:

$$F(x,y) = Dx^2(H-y) + G(H-y)^3 + \sum_{n=1}^{\infty} \cos\alpha x\left(C_1\cosh\alpha y + C_2\sinh\alpha y + C_3 y\cosh\alpha y + C_4 y\sinh\alpha y\right) \qquad (2a)$$

where D, G, C_1, C_2, C_3 and C_4 are constants to be determined by proper boundary conditions.

In order to satisfy the equilibrium condition, the stresses are expressed as follows:

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}$$

$$= 6G(H-y) + \sum_{n=1}^{\infty} \cos\alpha x \left[C_1\alpha^2 \cosh\alpha y + C_2\alpha^2 \sinh\alpha y + C_3\alpha(2\sinh\alpha y + \alpha y \cosh\alpha y) + C_4\alpha(2\cosh\alpha y + \alpha y \sinh\alpha y) \right], \quad (3a)$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2}$$

$$= 2D(H-y) - \sum_{n=1}^{\infty} \alpha^2 \cos\alpha x \left(C_1 \cosh\alpha y + C_2 \sinh\alpha y + C_3 y \cosh\alpha y + C_4 y \sinh\alpha y \right), \quad (4a)$$

$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} + wx$$

$$= wx + \sum_{n=1}^{\infty} \alpha \sin\alpha x \left[C_1\alpha \sinh\alpha y + C_2\alpha \cosh\alpha y + C_3(\cosh\alpha y + \alpha y \sinh\alpha y) + C_4(\sinh\alpha y + \alpha y \cosh\alpha y) \right]. \quad (5a)$$

where w is weight per unit volume of the girder.
The boundary conditions for this case are:

$$\left.\begin{aligned} y = +H \qquad & \tau_{xy} = 0, \qquad \sigma_y = 0, \\ y = -H \qquad & \tau_{xy} = 0, \quad \sigma_y = -2wH - \frac{4WHL}{\pi C} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\alpha C \cos\alpha x, \end{aligned}\right\} \quad (6b)$$

and,

$$\int_{-L}^{L} (\sigma_x - \nu\sigma_y)\, dx = 0.$$

The constants in eqs. (2a) are found to be:

$$\left.\begin{aligned} C_1 &= (-1)^n \frac{4WH \sin\alpha C}{\alpha^3 C} K_1, \\ C_2 &= (-1)^{n+1} \frac{4WH \sin\alpha C}{\alpha^3 C} K_2, \\ C_3 &= (-1)^n \frac{4WH \sin\alpha C}{\alpha^2 C} K_3, \\ C_4 &= (-1)^{n+1} \frac{4WH \sin\alpha C}{\alpha^2 C} K_4, \\ D &= -\frac{W}{2} \qquad \text{and} \qquad G = -\frac{\nu W}{6}. \end{aligned}\right\} \quad (13)$$

It has been shown[7] that for girders on roller supports the result can be obtained by superposition of two cases: (a) a girder, subjected to its own weight, is supported continuously along its entire lower edge, and (b) the same girder, resting on equidistant supports, is loaded at its lower edge by an uniformly distributed load equal to the weight of the girder. Since the supports of the girder in case (a) experience longitudinal displacement, it should not confused with the present case of fixed supports.

The Bending Stresses

For illustration and comparison, the bending stresses at the centerline of span and the centerline of support due to several loading conditions are plotted. The height-to-span ratio is taken as one, and the width of the supports, one-tenth of the span length. The curves are shown in Figs. 7 to 10.

In certain cases, the bending stress curve created by body weight may be substantial. As shown in Figs. 7 and 8, the stress pattern at the top is equivalent to that of a weightless girder with its entire dead weight (q per linear length) hanging at the bottom, while the stress pattern at the bottom is equivalent to that of a weightless girder with its entire dead weight applied at the top. It is also noted that the stress curve due to body weight could have been interpolated from the curves for bottom- and top-loaded girders. At the mid-height, for instance, where half of the body weight is above as well as below, the bending stress falls precisely in the middle of the extreme cases. Further investigation shows that, when the girder is loaded uniformly over its entire length at an intermediate elevation, the bending stress curve will consist of the top part of the curve for the bottom-loaded girder, the bottom part of the curve for the top-loaded girder and a horizontal line joining the two curves at the elevation of the applied load.

For a top-loaded girder, the bending stresses at the centerline of span and the centerline of supports are plotted in Figs. 9 and 10 for the following cases: uniform load covering the entire span, uniform load covering partially the central span and a concentrated load at the centerline of span. It can be observed that, for a given girder carrying at its top a constant load, but uniformly distributed over three different lengths, the bending stresses at the lower part of the girder are more or less the same, although they show significant differences at the top part of the girder.

The curves shown in Fig. 11 are obtained by superimposing two known solutions. Let d equal to L-c in Fig. 1, reverse the load, then superimpose it on the load in Fig. 2, case "A" is obtained; if we let d equal to L/2, then case "B" results. It is obvious that many interesting cases may be obtained in the same manner.

The Shear Stresses

The distribution of shear stresses at quarter span due to uniform and concentrated loads at the top and bottom edge of the girder is illustrated in Fig. 12. Marked differences can be observed among the three curves themselves as well as from the usual concept of shear stresses in shallow beams.

7. "Theory of Elasticity", by Timoshenko and Goodier, 2nd Ed., 1951, McGraw-Hill Co. P. 53

Conclusions

It is well known that the stress distribution of a deep girder deviates considerably from that of a shallow beam. Following the present analysis, many new aspects concerning stresses in deep beams are developed, and the following conclusions may be drawn:

(a) In general, bending and shear stresses in a deep girder with roller supports are different for loads at different elevations. However, when the load is uniformly distributed over the whole span, the bending and shear stresses in a girder loaded at the top will be identical with those in the same girder, but loaded at the bottom.

(b) In general, bending and shear stresses in a deep girder with fixed supports are different for loads at different elevations. But, when the load is uniformly distributed over the entire span, the shear stresses become independent of load elevations.

(c) In general, bending and shear stresses in a girder with roller supports are different from those with fixed supports under identical loadings. But, when loads are applied at the bottom edge, bending and shear stresses are same whether the girder is on roller or fixed supports.

For most problems, it is only practical to emphasize on the bending stress τ_x and the shear stress σ_{xy}. In some cases, the principal stresses may be important. In concrete designs (6) (8), for instance, the diagonal tension depends also on σ_y.

With the load applied at the top, the condition of fixed supports imposes an additional uniform compression to the bending stresses throughout the girder. Therefore, the summation of bending stresses at any section will not be zero, but equal to a constant horizontal compressive force $\frac{2\nu qHd}{L}$. If the girder is supported on a series of flexible columns but rigidly built in at the ends in the sense that any longitudinal displacement is prevented, then two equal and opposite horizontal reactions, equal to $\frac{2\nu qHd}{L}$, will be created in the built-in ends. Consequently the interior columns will experience neither longitudinal displacement, nor horizontal reaction. The ideal boundary conditions for fixed supports are thus achieved. It follows that for a long continuous girder on sufficiently rigid supports, similar conditions will prevail if the span considered is sufficiently away from the end spans.

The results summarized here are exact for typical interior span of a long continuous girder with height-to-span ratio of one-half or more. Although the results are applicable to any kind of construction material, the main practical emphasis however, is on reinforced concrete construction. It is with this in mind that the girder with fixed supports is treated, and the girder subjected to its own weight is presented.

It is hoped that this work would be a step further toward completion of the deep girder analysis. Inasmuch as any theoretical work enjoys its sanctuary only within the limits of its assumptions, its application to individual cases has to be left to professional sound judgment.

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8. See also: "Bemessung Wandartiger Trager", by H. Bay, Beton und Stahlbeton, Vol. 47, 1952, pp. 54-57

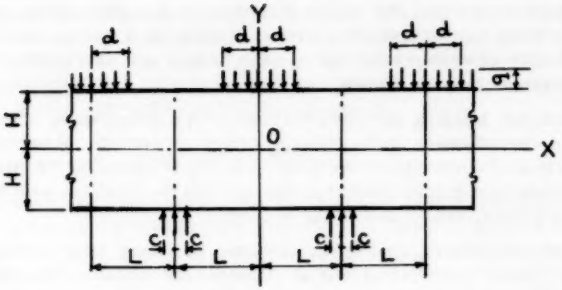

FIG. 1

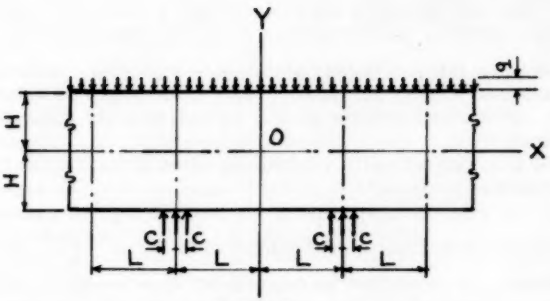

FIG. 2

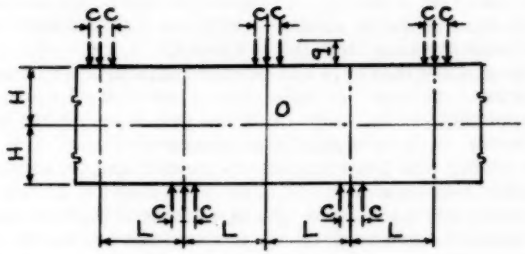

FIG. 3

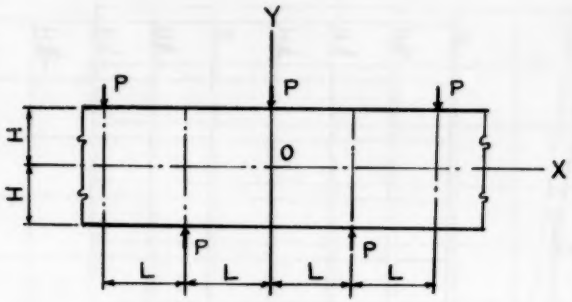

FIG. 4

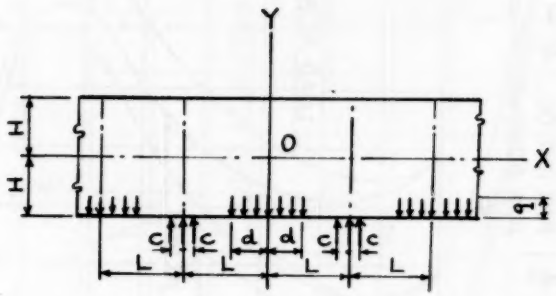

FIG. 5

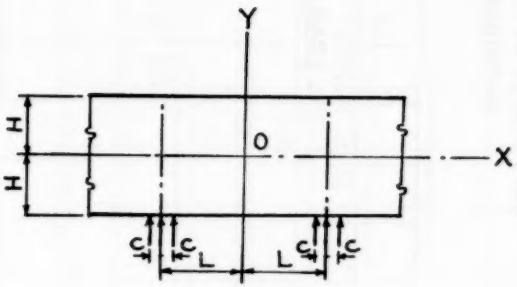

FIG. 6

BENDING STRESSES AT ℄ OF SUPPORT DUE TO UNIFORM LOAD

UNIT STRESS IN TERM OF $\frac{q}{b}$, FOR RATIOS: $\frac{H}{L}=1$, $\frac{c}{L}=0.1$, $\nu=0.2$
b=GIRDER WIDTH FOR BODY WEIGHT, $q=2WH$

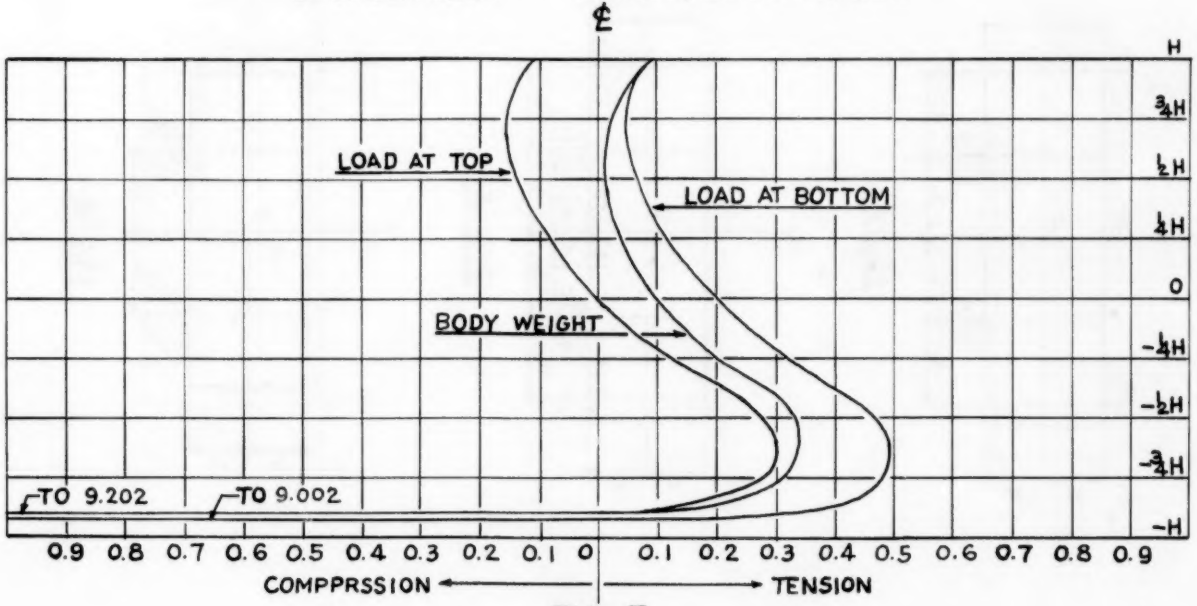

FIG. 7

BENDING STRESSES AT ℄ OF SPAN DUE TO UNIFORM LOAD

UNIT STRESS IN TERM OF $\frac{q}{b}$, FOR RATIOS: $\frac{H}{L}=1$, $\frac{c}{L}=0.1$, $\nu=0.2$

b=GIRDER WIDTH FOR BODY WEIGHT $q=2WH$

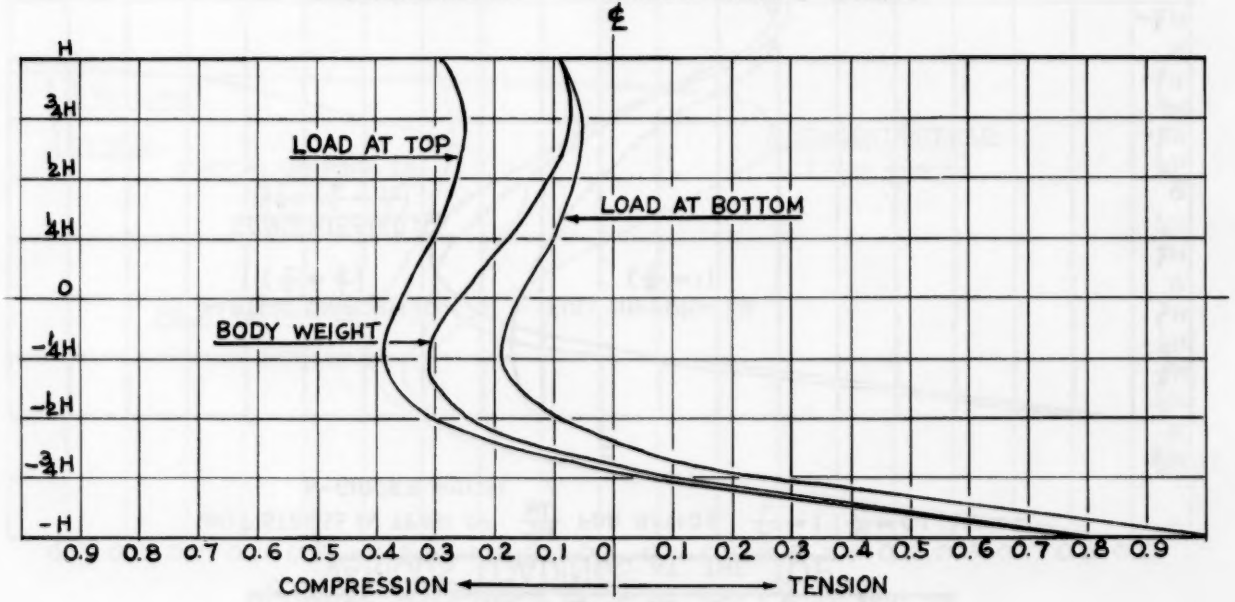

FIG. 8

BENDING STRESSES AT ℄ OF SUPPORT DUE TO VARIOUS LOADINGS AT THE TOP

UNIT STRESS IN TERM OF $\frac{qd}{bL}$ FOR RATIOS: $\frac{H}{L}=1$ $\frac{c}{L}=0.1$ $\nu=0.2$

b-GIRDER WIDTH

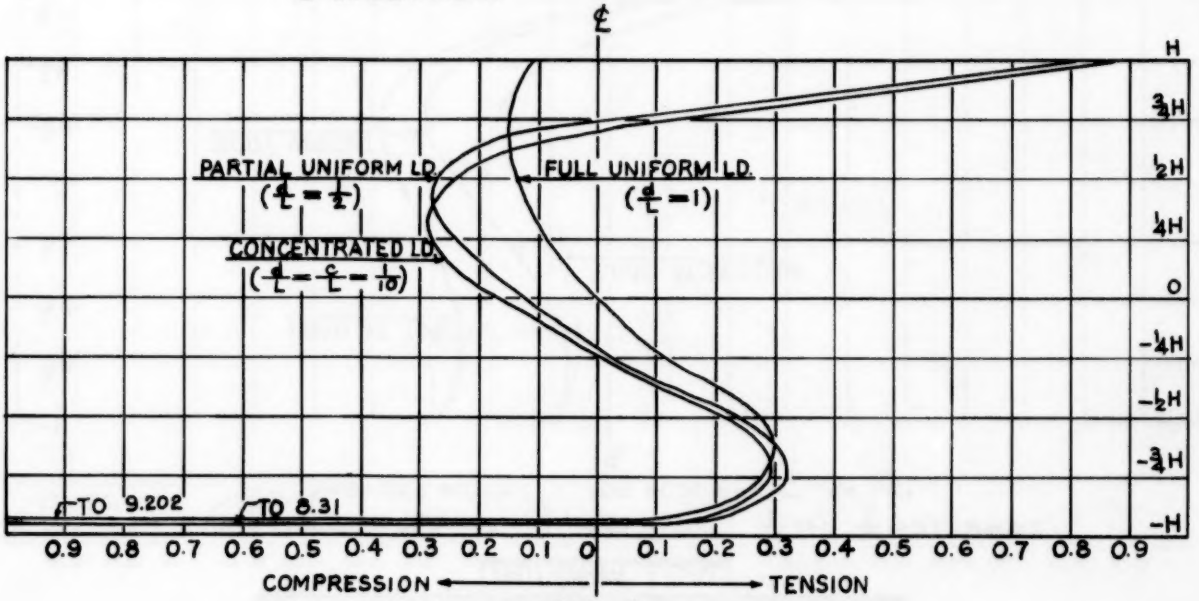

FIG. 9

BENDING STRESSES AT ℄ OF SPAN DUE TO VARIOUS LOADINGS AT THE TOP

UNIT STRESS IN TERM OF $\frac{qd}{bL}$ FOR RATIOS: $\frac{H}{L}=1$, $\frac{C}{L}=0.1$, $\nu=0.2$

b=GIRDER WIDTH

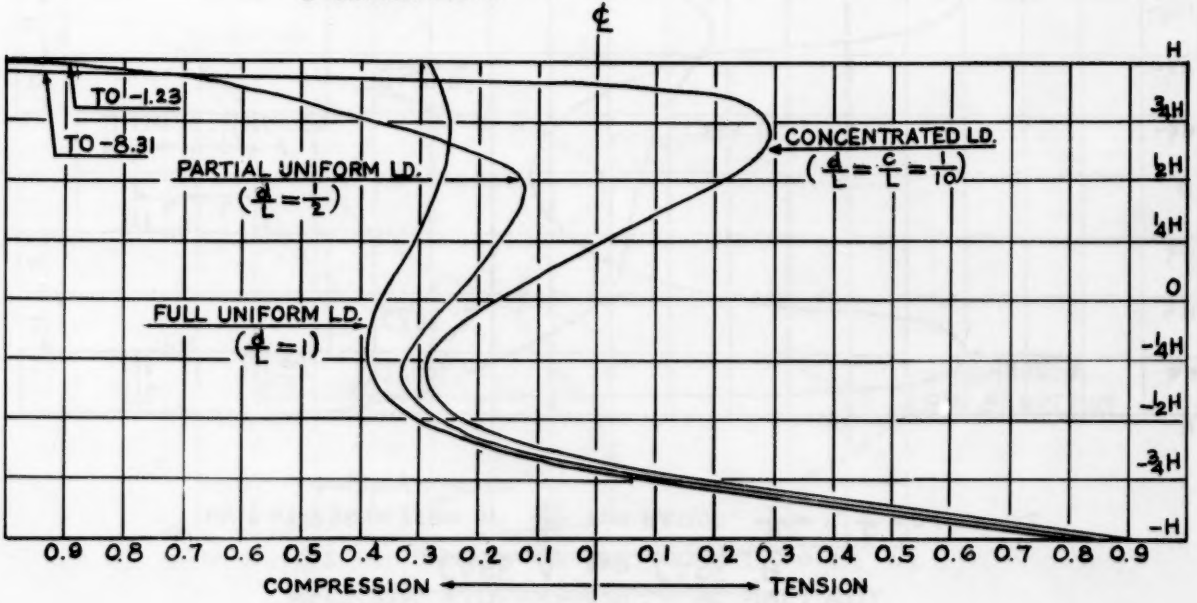

FIG. 10

BENDING STRESSES AT ℄ OF SUPPORT
CASE "A" AND CASE "B"

UNIT STRESS IN TERM OF $\frac{qd}{bL}$ FOR RATIOS; $\frac{H}{L}=1$, $\frac{c}{L}=0.1$, $\nu=0.2$

b=GIRDER WIDTH

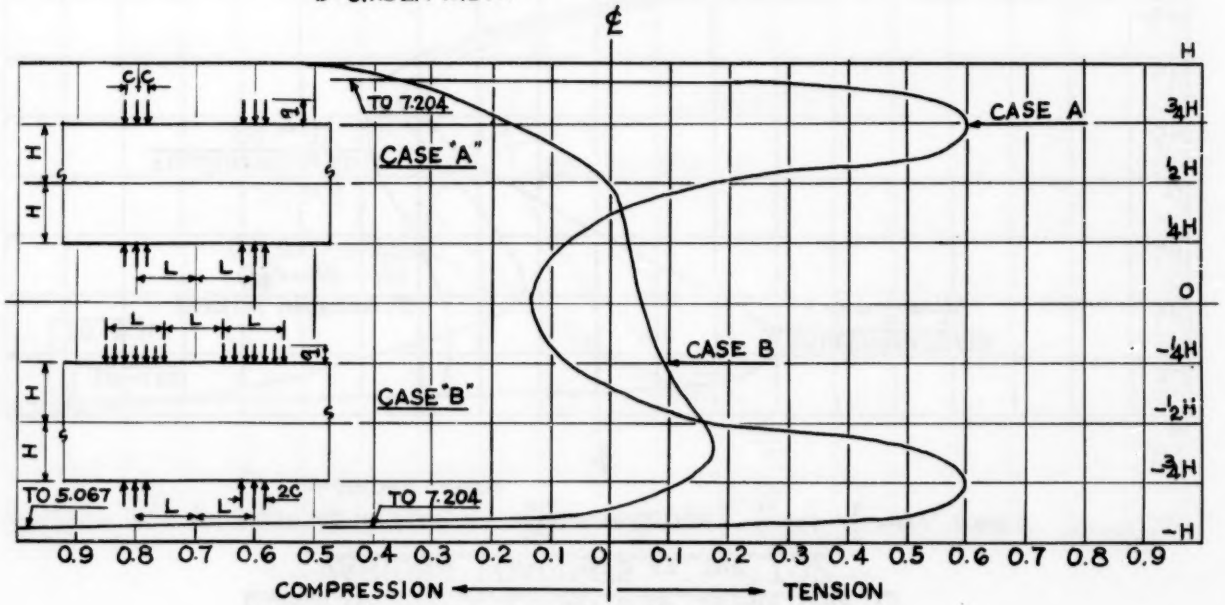

FIG. 11

SHEAR STRESSES AT QUARTER SPAN DUE TO UNIFORM AND CONCENTRATED LOADS AT TOP AND BOTTOM

UNIT STRESS IN TERM OF $\frac{qd}{bL}$, FOR RATIOS: $\frac{H}{L}=1$, $\frac{c}{L}=0.1$, $\nu=0.2$

b = GIRDER WIDTH

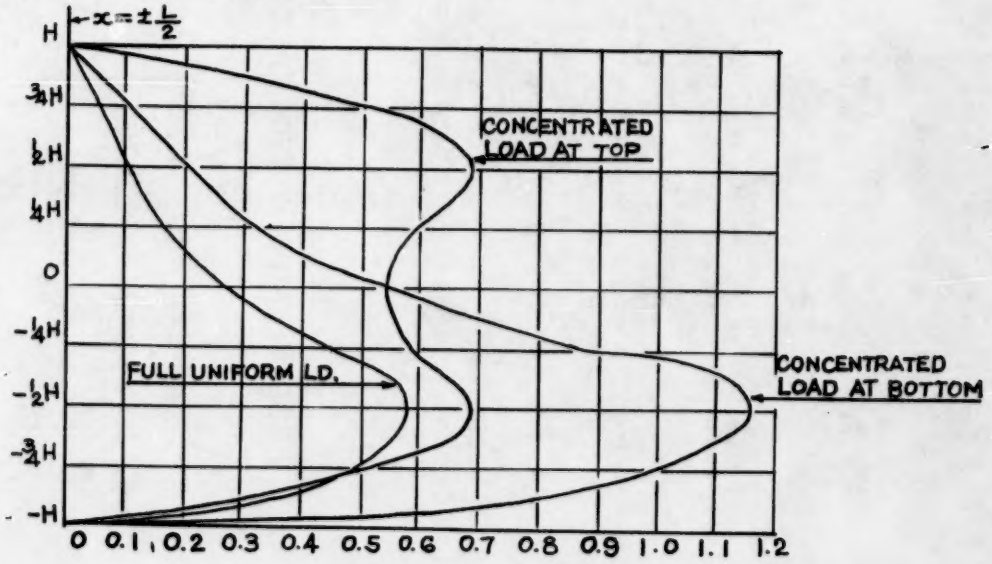

FIG. 12

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